Homogenization of Metrics in Random Structures and Applications to Materials Science

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Abstract

Localization phenomena in heterogeneous materials occur when the main variables that govern the material's physical response exhibit abrupt discontinuities in space. Common examples in mechanics include cracks in brittle fracture or shear bands in plasticity. These problems are characterized long-range correlations and a strong interplay between the microstructure and the material's effective (i.e. overall) response. In the present work, we focus on a model problem of a class of strongly-nonlinear conductors, and establish rigorous relationships between the microstructure, its effective response and the localization patterns of the electric or current field. Using random set theory and Fourier-based numerical predictions, we examine the role of various random and periodic binary microstructures on the material effective properties. We also investigate the length of minimal paths and volume of minimal hyperplanes in arbitrary dimension. We show that in the case of a dilute (i.e. small) volume fraction of one of the two phases, the length of minimal paths depends in general in a non-analytical way on the volume fraction of the dilute phase. We give a geometrical interpretation of the exponents.

References

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